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Hydromagnetic surface waves along a compressible cylindrical plasma column surrounded by neutral gas

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Abstract

The characteristics of hydromagnetic surface waves propagating along the interface of a compressible cylindrical plasma column surrounded by neutral gas embedded in a steady magnetic field along the axis of the cylinder are studied by varying the ratio of the magnetic field and the relative density of the two media. Symmetric and asymmetric modes are found to exist. When Alfven speed ‘\(v_A\)’ is less than the sound speed ‘\(s\)’ in the neutral gas the symmetric mode suffers lower cut-off in \(k\) for propagation. The cut-off window is narrowed when the magnetic field in the neutral gas is increased. For \(v_A > s\), the density of the neutral gas is found to suppress, restrict to a certain window, and allow the free propagation of both modes. There exists a critical magnetic field ratio for certain environments, in which both slow and fast surface waves are present simultaneously in symmetric and asymmetric modes. The findings for a special case are found to be in agreement with those obtained for an isolated magnetic flux tube studied earlier.

1. Introduction

In laboratory plasmas such as magnetically confined fusion plasmas or low-pressure arc discharges, the situation arises where plasma is enveloped by neutral gas. Alfven and Smars (1960) have proposed the idea that thermonuclear plasma can be confined by a cold gas blanket if the heat conduction in the ionized transition region is sufficiently reduced by a strong magnetic field. They have remarked that the gas blanket provides a barrier against impurities from the walls and that a plasma confined in this way is not subject to the same types of instabilities as a plasma confined by magnetic fields in vacuum. Braams (1966) has stressed the advantages of a gas blanket with an axial magnetic field. He has stated that steady-state plasma, confined by a cold-gas blanket in a toroidal magnetic field, is stable against magnetohydrodynamic instabilities. Ohlsson (1977) investigated the joint effects of interaction of plasma--neutral gas and resistivity from the stability point of view.

Other studies of plasma in a neutral gas environment are the numerical simulation of neutral gas release experiments in the ionosphere by Okuda and Choueiri (1994), and the plasma flow through the gas cloud in a tokamak divertor by Kraserninnikov and Soboleva.
Parameters such as the density and neutral temperature of magnetized helium plasma columns submerged in a cool neutral gas have been studied by Chiu and Cohen (1996).

The interface formed due to the change in magnetic field or density is found to support hydromagnetic surface waves (Hasegawa and Uberoi 1982, Edwin and Roberts 1981, Somasundaram et al 1999). The study of hydromagnetic surface waves along the interface of two media helps us to understand the stable wave propagating along it. Evidence for Alfven surface waves in the laboratory was provided by Lehane and Paoloni (1972). Hydromagnetic surface wave propagation along an incompressible cylindrical plasma surrounded by a neutral gas has been studied by Uberoi and Somasundaram (1982a). The acoustic surface-wave generation along an incompressible cylindrical plasma column surrounded by a compressible gas has been studied by Uberoi and Somasundaram (1982b). In this paper, we study the effect of compressibility on the surface wave propagation along a cylindrical plasma column surrounded by a neutral gas embedded in a steady magnetic field, lying along the axis of the cylinder. We begin our study in identifying the possible regions for the surface wave propagation from the dispersion equation. Then, the dependence of surface wave characteristics with the ratio of steady magnetic field, the relative densities of the two media, and the temperatures of the media are studied. It is found that the surface wave propagation is significantly affected when the magnetic field in the neutral gas is less than that in the plasma. Also, the change in temperature introduces a lower cut-off value in the wave vector $k$ for symmetric surface wave (sausage-type) propagation. Interestingly, a special case of our result for a compressible plasma embedded in a magnetic field surrounded by a neutral gas corresponds to the result obtained for an isolated flux tube surrounded by a field free media by Evans and Roberts (1990). Dispersion characteristics for slow and fast modes for such an environment have also been studied.

2. The geometry

Consider a compressible cylindrical plasma column of radius '$a$' with mass density $\rho_o$ embedded in a magnetic field $B_o$ surrounded by compressible gas of mass density $\rho_g$ and magnetic field $B_g$. Both $B_o$ and $B_g$ are directed along the $Z$-axis, i.e. the axis of the column (figure 1).

3. Dispersion relation

Using the linearized hydromagnetic equations for a compressible and infinitely conducting plasma (Roberts 1981, Pu and Kivelson 1983) and for perturbation of the form $f(r, \varphi, z, t) = f(r) \exp (i(kz + \ell \varphi - \omega t)$ we get,

$$z^2 \frac{\partial^2 p}{\partial z^2} + z \frac{\partial p}{\partial z} - p(z^2 + \ell^2) = 0 \quad (3.1)$$
where \( p \) is the perturbed pressure, \( z = nr \) and,

\[
n = k \sqrt{\frac{(v_A^2 - (\omega^2/k^2))(c^2 - (\omega^2/k^2))}{(c_T^2 - (\omega^2/k^2))(c^2 + v_A^2)}}
\]

(3.2)

\( c_T^2 = \frac{c^2 v_A^2}{c^2 + v_A^2} \).

Here \( c \) is the sound velocity in plasma and \( v_A \) is the Alfvén wave velocity.

Equation (3.1) is a modified Bessel equation for which the solution is given by,

\[
p = D I_\ell(nr) \quad r < a
\]

(3.3)

where \( D \) is an arbitrary constant and \( I_\ell \) is a modified Bessel function of order \( \ell \).

By reducing the linearized hydrodynamic equations for the neutral gas (Uberoi and Somasundaram 1982a, b) and for perturbation of the form \( f(r, \varphi, z, t) \equiv f(r) \exp (kz + i \varphi - \omega t) \) we get,

\[
\left( \nabla^2 + \frac{\omega^2}{s^2} \right) p_g = 0.
\]

(3.4)

The solution of equation (3.4) is given by,

\[
p_g = E K_\ell(\tau r) \quad r > a
\]

(3.5)

where \( K_\ell(\tau r) \) is the modified Bessel function of order \( \ell \), \( E \) is an arbitrary constant and

\[
\tau = k \sqrt{1 - \frac{\alpha^2}{s^2 k^2}}
\]

As there is no conduction current in the gas and the displacement current is neglected, the perturbed magnetic field in gas is given by \( \nabla \times b_g = 0 \), which means that \( b_g \) can be derived from a magnetic potential \( \psi \), i.e.

\[
b_g = \nabla \psi
\]

(3.6)

and since \( \nabla \cdot b_g = 0 \), \( \psi \) obeys the equation,

\[
\nabla^2 \psi = 0.
\]

(3.7)

Solving equation (3.7) we get,

\[
\psi = F K_\ell(kr) \quad r > a
\]

(3.8)

where \( K_\ell(kr) \) is the modified Bessel function of order \( \ell \) and \( F \) is an arbitrary constant.

Applying the boundary conditions (Uberoi and Somasundaram 1982a, b), that

(a) the tangential component of the electric field seen by the plasma just above the surface must vanish,

(b) the total pressure is continuous,

(c) the normal velocity component is continuous at the boundary \( r = a \),

we get the dispersion relation:

\[
x^2 \left[ \eta h_\ell(\eta)(\tau a) \right] + \frac{k}{n} - \left[ \beta^2 F_\ell([k, n], a) + \frac{k}{n} \right] = 0
\]

(3.9)

where

\[
h_\ell(\eta) = \frac{I_\ell(\eta)}{I_\ell(\eta)} \quad g_\ell(\tau a) = -\frac{K_\ell(\tau a)}{K_\ell(\tau a)} \quad F_\ell([k, n], a) = -\frac{I_\ell(\eta) K_\ell(\eta)}{I_\ell(\eta) K_\ell(\eta)}
\]

\( x = \omega/k v_A \) is the normalized phase velocity, \( \alpha = v_A/s, \eta = \rho_g/\rho_o \) and \( \beta = B_g/B_o \) are the interface parameters. The prime stands for differentiation with respect to \( r \).
4. Discussion

In the incompressible limit, $c \to \infty$ and $n \to |k|$, $F(\{k,n\}a) \to F(\{k\}a) = F(ka)$ and hence, for the incompressible case the dispersion relation (3.9) becomes,

$$x^2 \left[ \eta h_n(ka) g_1(\tau a) \right] - [\beta^2 F(ka) + 1] = 0$$

(4.1)

as obtained by Uberoi and Somasundaram (1982a). They have discussed this dispersion relation at length.

When vacuum surrounds the plasma column instead of neutral gas the dispersion relation reduces to:

$$\beta^2 F(\{k,n\}a) + \frac{k}{n} (1 - x^2) = 0.$$  

(4.2)

The normalized phase velocity in this case is given by,

$$x = \sqrt{\frac{\beta^2 F(\{k,n\}a) + 1}{k}}.$$  

(4.3)

For an incompressible plasma $F(\{k,n\}a) \to F(ka)$, equation (4.3) reduces to,

$$x = \sqrt{1 + \beta^2 F(ka)}.$$  

(4.4)

Equation (4.4) has been discussed by Uberoi and Somasundaram (1980).

When the magnetic field in the neutral gas is absent, $B_g = 0$, i.e. $\beta^2 = 0$, equation (3.9) reduces to,

$$x^2 \left[ \eta h_n(na) g_1(\tau a) \right] - \frac{k}{n} = 0.$$  

(4.5)

The dispersion relation given in equation (4.5) is the same as that found for a cylindrical plasma column embedded in a magnetic field and surrounded by a field free plasma, as obtained by Evans and Roberts (1990, equation (2.11)). Result (4.5) can also be obtained from the result of Somasundaram et al (1999) by taking the flow velocity as zero in their equation (4.1).

At equilibrium, the pressure balance condition demands

$$P_g + \frac{B_g^2}{2\mu_o} = P + \frac{B_n^2}{2\mu_o}.$$  

(4.6)

Rearranging equation (4.6) we get the normalized sound speed in the plasma column as:

$$c^2 = \frac{\eta}{\alpha^2 + \frac{\gamma}{2} (\beta^2 - 1)}$$  

(4.7)

where $c = \sqrt{(\gamma P)/\rho}$ is the sound speed in plasma, $\alpha = v_A/s$, where $s = \sqrt{(\gamma P_g)/\rho}$ is the sound speed in the neutral gas and $\gamma$ is the ratio of the specific heats, taken to be 1.66.

For a surface wave, $n^2$ has to be positive. For $n^2$ to be positive, the following inequalities are to be satisfied,

$$\frac{\omega}{k} < c_T$$  

(4.8)

or

$$\text{min}(c, v_A) < \frac{\omega}{k} < \text{max}(c, v_A).$$  

(4.9)

The inequalities represented by equations (4.8) and (4.9) divide the propagation zone of the hydromagnetic surface waves into possible and forbidden zones. Further, in order for the factor $\sqrt{1 - \alpha^2 x^2}$ to be real in dispersion relation (3.9), the following condition is imposed:

$$0 < \frac{\omega}{k} < s$$  

(4.10)
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i.e. phase velocity is less than sound velocity in neutral gas.

From equation (4.7) we note that for any given \( \eta \) and \( \beta^2 < 1 \), there is a critical value \( \alpha (\alpha_c) \) above which \( c/v_A \) becomes negative and the equilibrium pressure balance condition is not satisfied. The equation for the critical value of \( \alpha \) is given by,

\[
\alpha_c = \sqrt{\frac{2\eta}{\gamma(1 - \beta^2)}}.
\] (4.11)

To find the possible regions, we plot three curves (solid curves) by setting the three factors in equation (3.2) to zero and plotting \( 1/\alpha \) in figure 2(a) for \( \beta^2 = 0 \), \( \eta = 1 \), and in figure 2(b) for \( \beta^2 = 1 \) and \( \eta = 1.5 \). Regions satisfying the surface wave conditions given in inequalities (4.8)–(4.10) are marked as ‘P’, indicating possible regions. Subscripts \( \text{s} \) and \( \text{f} \) denote the regions for slow and fast surface waves, respectively. In figure 2(a) we observe that there are two possible regions which are

(i) region \( P_s \) at which the phase velocity satisfies the inequality,

\[
\frac{\omega}{k} < \min(c_T, s)
\]

(ii) region \( P_f \) at which the phase velocity satisfies the inequality,

\[
c < \frac{\omega}{k} < \min(s, v_A), \text{ or } v_A < \frac{\omega}{k} < \min(c, s).
\]

We also superimpose the dispersion curves (dashed curves) obtained for \( k\alpha = 1 \) on figures 2(a) and (b). We note from figure 2(a) that, for \( 0 < \alpha < 1 \), surface waves are found to exist only in the slow mode region, and for \( 1 < \alpha < \alpha_c \), only fast waves are found to exist. Similarly, from figure 2(b) we observe that both regions \( P_s \) and \( P_f \) exist for \( \alpha < 1 \), but only \( P_s \) exists for \( \alpha > 1 \). However, for all values of \( \alpha \) only slow surface waves are found to exist.

To find the possible regions for surface wave propagation, three different cases are shown in figures 3(a)–(c): the first with \( \eta = 1.5, \beta^2 = 1 \) (also applicable to the case \( \beta^2 = 0–2 \)), \( \alpha = 0.2 \); the second with \( \eta = 1.5, \beta^2 < 1 \) and \( \alpha = 1.2 \); and the third with \( \eta = 1.5, \beta^2 > 1 \) and \( \alpha = 1.2 \).

From figure 3(a) we note that when \( v_A < s \) possible regions for both the fast and slow surface waves exist, irrespective of the magnetic field ratio. From figures (b) and (c) we observe that when \( v_A > s \), two cases arise: (i) for \( \beta^2 < 1 \), possible regions for both the fast and slow surface waves exist, and (ii) for \( \beta^2 > 1 \), a possible region for the slow surface waves only exists.

4.1. The effect of magnetic field

To study the effect of variation of magnetic field, we plot the dispersion curves obtained from equation (3.9) in figure 4(a), for \( \alpha = 0.2, \eta = 0.5 \) and \( \beta^2 = 0.0, 0.4, 1.0 \) and 2.0. The symmetric (\( \ell = 0 \)) modes are represented by solid curves and the asymmetric (\( \ell = 1 \)) modes by dashed curves. When \( \beta^2 = 0.0 \) and 0.4 (\( \beta^2 < \eta \)), there is only slow surface wave propagation for both symmetric and asymmetric modes. For small values of \( k\alpha \), the phase speed of the symmetric wave mode is higher than that for the asymmetric wave mode. As \( k\alpha \) increases the phase speed of the symmetric mode decreases while that of the asymmetric mode increases. For \( k\alpha > 1 \), the phase speeds of both modes become equal and represent the surface wave characteristics of a single interface. For \( \beta^2 = \eta \), there is no surface wave propagation. This behaviour is similar to uniform media without a discontinuity at the interface; it is notable as the interface between the neutral gas and plasma behaves as though there is no discontinuity. For \( \beta^2 > \eta \), there are two branches for the symmetric mode and one branch for the asymmetric
Figure 2. (a) Possible regions for surface wave propagation for $\beta^2 = 0.5$ and $\eta = 1.0$. Regions that satisfy conditions (4.8)–(4.10) are marked as $P_s$ (for slow surface waves) and $P_f$ (for fast surface waves). Dashed curves represent the dispersion curves for $ka = 1.0$. Note that the dispersion curves represent slow and fast surface waves lying in the possible regions $P_s$ and $P_f$, respectively. 
(b) As (a) but for $\beta^2 = 1.0$ and $\eta = 1.5$. The dashed curve represents the dispersion curve for $ka = 1.0$ which is always a slow surface mode. Note that the dispersion curve lies only in the slow propagation region.
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Figure 3. Schematic diagram (not to scale) representing the possible regions for surface wave propagation for (a) $\eta = 1.5$, $\beta^2 = 1.0$ (also applicable to the case $\beta^2 = 0-2$), $\alpha = 0.2$ ($c_T < v_A < s < c$); (b) $\eta = 1.5$, $\beta^2 < 1.0$, $\alpha = 1.2$ ($c_T < c < s < v_A$); and (c) $\eta = 1.5$, $\beta^2 > 1.0$, $\alpha = 1.2$ ($c_T < s < v_A < c$).

mode. Both the branches of the symmetric mode suffer a lower cut-off value $k = k_c$ for propagation. Thus, the symmetric mode propagation is restricted for a wavelength region $0 < \lambda < \lambda_c$ ($\lambda_c$ represents the cut-off wavelength). As $\beta^2$ is increased (the magnetic field in the neutral gas is increased), the cut-off value $k_c$ decreases ($\lambda_c$ increases). The dispersion
curve with $\beta^2 = 0$ is similar to the curve obtained for an isolated magnetic flux tube by Evans and Roberts (1990). Thus, we note that a plasma cylinder with a magnetic field surrounded by a neutral gas or by a plasma behaves identically for surface waves.

Figures 4(b) and (c) show the dispersion curves for $\alpha = 0.2$, $\beta^2 = 0.0, 0.5, 1.0, 1.5$ and 2.0, and $\eta = 1$ and 1.5, respectively. Comparing figures 4(a)–(c) we note that for $\beta^2 < \eta$ only slow surface waves exist and for $\beta^2 > \eta$ only fast surface waves exist. Also, we note that as the value of $\eta$ is increased (the density of the neutral gas is increased), for any given $\beta^2$ value, the cut-off of $k$ for the symmetric mode is decreased and makes the free propagation of the symmetric mode. In all three cases, we note that for $\beta^2 < \eta$, both symmetric and asymmetric modes have no cut-off. For $\beta^2 > \eta$ there are two branches for the symmetric mode, one of which transforms to a body wave as $ka$ increases. Further, the symmetric modes are restricted for a wavelength region $0 < \lambda < \lambda_c$. The case represented in figure 4(b) for $\beta^2 = 0$ nearly approximates the case (i) discussed by Evans and Roberts (1990).

### 4.2. The effect of temperature

The factor $\alpha = v_A/s$, can be taken to represent the temperature of the neutral gas, as $s \propto \sqrt{P}$, and $P \propto \text{temperature}$, where $P$ is the pressure in the neutral gas. An increase in $\alpha$ means a decrease in temperature of the neutral gas. We increased the value of $\alpha$ to 1.2 and plotted dispersion curves for $\beta^2 = 0.0, 0.5, 1.0, 1.5$ and 2.0 with $\eta = 0.5$ in figure 5(a), $\eta = 1.0$ in figure 5(b) and $\eta = 1.5$ in figure 5(c). Solid curves represent symmetric modes and dashed curves show the asymmetric mode. From figure 5(a), we note that fast surface waves exists only for $\beta^2 = 1.0$. For $\beta^2 > 1.0$, there are no surface waves and for $\beta^2 < 1.0$ the value of $\alpha$ becomes greater than $\alpha_c$ where the equilibrium pressure balance condition is not satisfied. Therefore, such an environment is physically not possible. This behaviour occurs only for $\eta = 0.5$, i.e. when the density of the neutral gas is lower than the inner plasma density.

We note from figure 5(b), that for $\eta = 1.0$, i.e. when the densities of the neutral gas
Figure 4. Dispersion curves for (a) $\alpha = 0.2$, $\eta = 0.5$ and $\beta^2 = 0.0, 0.4, 1.0, 1.5$ and 2.0; (b) $\alpha = 0.2$, $\eta = 1.0$ and $\beta^2 = 0.0, 0.5, 1.5$ and 2.0; (c) $\alpha = 0.2$, $\eta = 1.5$ and $\beta^2 = 0.0, 0.5, 1.0, 1.5$ and 2.0. Solid curves represent symmetric waves ($\ell = 0$) and dashed curves represent asymmetric waves ($\ell = 1$).
and the plasma are equal, and for $\beta^2 = 0.5$ fast surface waves for both symmetric and asymmetric modes exist for all $ka$ values. However, for $\beta^2 = \eta$, there are no surface waves. For $\beta^2 > \eta$ slow surface waves for both symmetric and asymmetric modes are present, but with a lower cut-off value of $k$. The cut-off values of $k$ decrease with an increase in $\beta^2$. For $\beta^2 = 2.0$, surface waves for the symmetric mode exist for all values of $k$ but for the asymmetric mode there is still a cut-off value for $k$. Figure 5(c) gives the dispersion curves for the case when $\eta = 1.5$, i.e. the density of the neutral gas is higher than the plasma density. An interesting feature we observe for $\beta^2 = 0.0$ and 0.5 is that for both symmetric and asymmetric modes the fast and slow surface waves are present simultaneously. For $\beta^2 = 1.0, 1.5$ and 2.0, the fast surface wave disappears and only slow surface waves exist both for symmetric and asymmetric modes. The case discussed in figure 5(c) for $\beta^2 = 0.0$ represents case (ii) of Evans and Roberts (1990) in which both fast and slow surface waves exist for both symmetric and asymmetric modes.

5. Conclusion

The interface between a cylindrical plasma surrounded by neutral gas and embedded in an axial magnetic field has been studied by assuming the plasma is compressible. The dispersion equation has been analysed and possible regions for surface wave propagation have been identified. These possible regions can be used as a ready reckoner to check whether a given environment can support a surface wave (slow or fast) or not. It has been found that the interface can support symmetric and asymmetric surface waves. When the Alfvén wave velocity in the plasma is less than the sound speed in the neutral gas ($v_A < s$) there is a lower cut-off value
Figure 5. Dispersion curves for (a) $\alpha = 1.2$, $\eta = 0.5$ and $\beta^2 = 1.0$; (b) $\alpha = 1.2$, $\eta = 1.0$ and $\beta^2 = 0.5, 1.2, 1.5, 1.8$ and 2.0; (c) $\alpha = 1.2$, $\eta = 1.5$ and $\beta^2 = 0.0, 0.5, 1.0, 1.5$ and 2.0. Note that for $\beta^2 < 1.0$, fast and slow surface waves are present simultaneously for both symmetric and asymmetric modes.
for the wave number $k$ for the symmetric mode and it is restricted to propagate in a wavelength window $0 < \lambda < \lambda_c$. The increase in magnetic field or density in the neutral gas relative to the same quantities inside the plasma reduces the cut-off window. When the square of the magnetic field ratio is greater than the mass density ratio, the symmetric mode is always accompanied by two branches, one of which transforms to body waves as $k$ increases. There is no such cut-off window in wave number when the plasma is taken as incompressible, as reported by Uberoi and Somasundaram (1982a). When the ratio of steady magnetic field and the mass density of the plasma and gas are equal, the interface behaves like a uniform medium. When the temperature is such that $v_A > s$, depending on the density of the neutral gas surrounding the hot plasma, the surface wave propagation can be suppressed when $\eta < 1$, restricted to a certain window of propagation in $k$ when $\eta = 1.0$, and both slow and fast surface waves can exist when $\eta > 1.0$. In general, it is found that when the magnetic field in the neutral gas is greater than the magnetic field in the plasma, only fast surface waves are found to exist when $v_A < s$ ($\alpha < 1$) and only slow surface waves are found when $v_A < s$ ($\alpha > 1$). Thus, the densities of the neutral gas and the plasma, and the magnetic field ratio can be used to control the presence of the surface waves along the interface. With $\beta^2 = 0.0$ our results are the same as those obtained for an isolated magnetic flux tube in the sunspot.

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