NOTE ON THE STABILITY OF PARALLEL MAGNETIC FIELDS

(Letter to the Editor)

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Abstract. The stability characteristics of parallel magnetic fields when fluid motions are present along the lines of force is studied. The stability criterion for both symmetric ($m = 0$) and asymmetric ($m = 1$) modes are discussed and the results obtained by Trehan and Singh (1978) are amended in the present study. The results obtained for the cylindrical geometry are shown to play an important role for $ka < 4$, where k is the wave number, a is the radius of the cylinder, compared to the results obtained by Geronicolas (1977) for the slab geometry.

1. Introduction

Recent studies on the stability of incompressible fluids with magnetic fields have aided much information both from a geophysical and astrophysical point of view (Geronicolas, 1977; Trehan and Singh, 1978). Trehan and Singh (1978) investigated the stability criterion for the flow in helical magnetic fields. They established that the configuration was unstable, when the velocity field exceeds certain critical value for the case when the velocity and magnetic fields are parallel. However, the stability criterion they obtained does not depict the real situation and some modifications are necessary to get a clear idea of the flow configuration. It is the purpose of this note to make the necessary amendments and discuss the stability criterion for the symmetric ($m = 0$) and asymmetric ($m = 1$) modes.

2. The Dispersion Relation

Consider a cylindrical column of incompressible, inviscid, and infinitely conducting fluid of mass density ρ_1 of radius a wherein the velocity and magnetic fields are parallel. This column of fluid is surrounded by an incompressible fluid of density ρ_2. For perturbations of the form $\exp\{i(kz + m\phi + \sigma t)\}$, the dispersion relation can be written (cf., Trehan and Singh, 1978; Equation (37)), as

$$\frac{\rho_2\sigma^2}{\rho_1[(\sigma + kV)^2 - k^2A^2]} = \frac{I_m(x)\, K_m'(x)}{I_m'(x)\, K_m(x)}\,, \tag{1}$$

where $x = ka$, I_m and K_m are the Bessel functions with imaginary argument of order m.

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3. Discussion

In this section we discuss the stability criterion for the symmetric ($m = 0$) and asymmetric ($m = 1$) modes. If we define

$$\eta = \rho_2/\rho_1 \ ,$$

$$F_m(x) = -\frac{I'_m(x)\, K_m(x)}{I_m(x)\, K'_m(x)} \ , \tag{2}$$

Equation (1) can be written as

$$\frac{\eta\sigma^2}{[(\sigma + kV)^2 - k^2A^2]} = -F_m(x) \ . \tag{3}$$

The roots of Equation (3) can be written as

$$\sigma = \frac{-k}{(1 + \eta F_m(x))}[V \pm \{A^2(1 + \eta F_m(x)) - \eta F_m(x) V^2\}^{1/2}]. \tag{4}$$

For instability to occur, the discriminant of Equation (4) should be negative which implies that

$$V^2 > A^2\left(1 + \frac{1}{\eta F_m(x)}\right). \tag{5}$$

Equation (5) is similar to the stability criterion obtained by Geronicolas (1977) for the slab geometry: namely,

$$V_G^2 > A^2\left(1 + \frac{1}{\eta \cot hkL}\right) \ , \tag{6}$$

where L is the half-width of the slab.

It is evident from Equation (5) that the stability criterion depends on the parameters η, $F_m(x)$ and m.

We give in Figure 1 the behaviour of the function $F_m(x)$ for $m = 0$ and $m = 1$. The limiting cases of $F_m(x)$ for $m = 0$ and $m = 1$ are as follows:

$$\begin{aligned} &F_0(x) \to 0 \quad &\text{as} \quad x \to 0 \ , \\ &F_0(x) \to 0.312 \quad &\text{as} \quad x \to 1 \ , \end{aligned} \tag{7}$$

and

$$\begin{aligned} &F_0(x) \to 1 \quad &\text{as} \quad x \to \infty \ . \\ &F_1(x) \to 1 \quad &\text{as} \quad x \to 0 \ , \\ &F_1(x) \to 0.729 \quad &\text{as} \quad x \to 1 \ , \end{aligned} \tag{8}$$

and

$$F_1(x) \to 1 \quad \text{as} \quad x \to \infty \ .$$

This shows that the stability criterion obtained by Trehan and Singh (1978) for

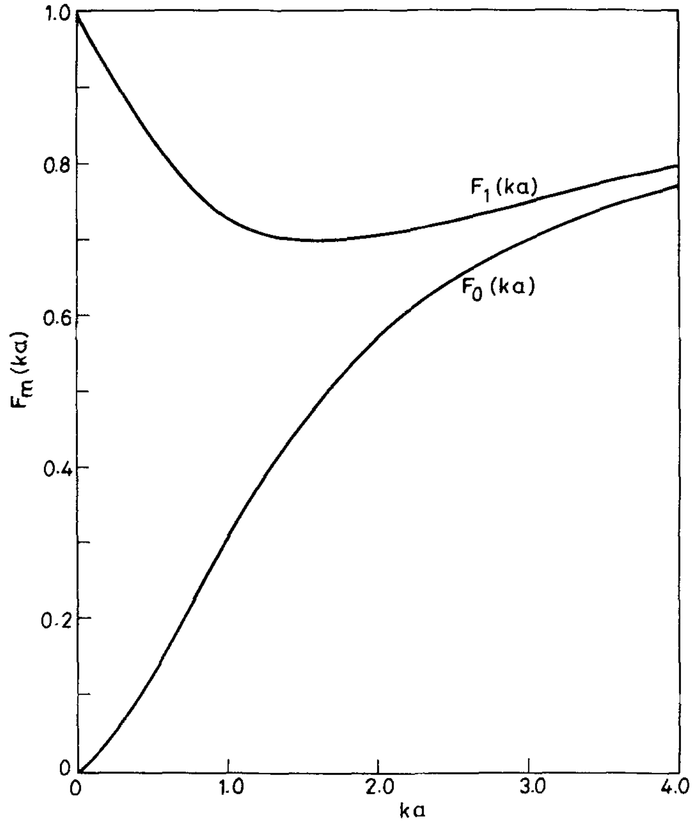

Fig. 1. Variation of the function $F_m(ka)$ with ka for $m = 0$ and $m = 1$.

the case when $x \simeq 1$ and $\eta \simeq 1$ is not true for the asymmetric mode ($m = 1$). Moreover, from Equation (7), it is clear that the stability criterion obtained by Trehan and Singh (1978) is not at all valid for the case when $m = 0$ and $x \to 0$ for $1/\eta F_0(x) \to \infty$.

In Figure 2, we give the neutral curves for $\eta = 0.5$, 1.0, 1.5, respectively, for the symmetric (solid lines) and asymmetric (dashed lines) perturbations. This gives a picture of how the stability regime is altered when the parameter η is changed. This leads to the conclusion that it is difficult to give a stability criterion which would decide the stability nature of the symmetric and asymmetric modes by a single relation. Thus, although the relation given in Equation (5) is a criterion for instability, it does not give the correct relation as it depends on the values the parameters m, $F_m(x)$ and η take. However, notice the fact that as $x \to \infty$, and $\eta = 1$, the behaviour of the function $F_m(x)$ is the same for $m = 0$ and $m = 1$, so that the neutral curves more or less coincide as can be seen from Figure 2. This means that for sufficiently large x, the stability criterion is of the form

$$V^2 > 2A^2 \, , \tag{9}$$

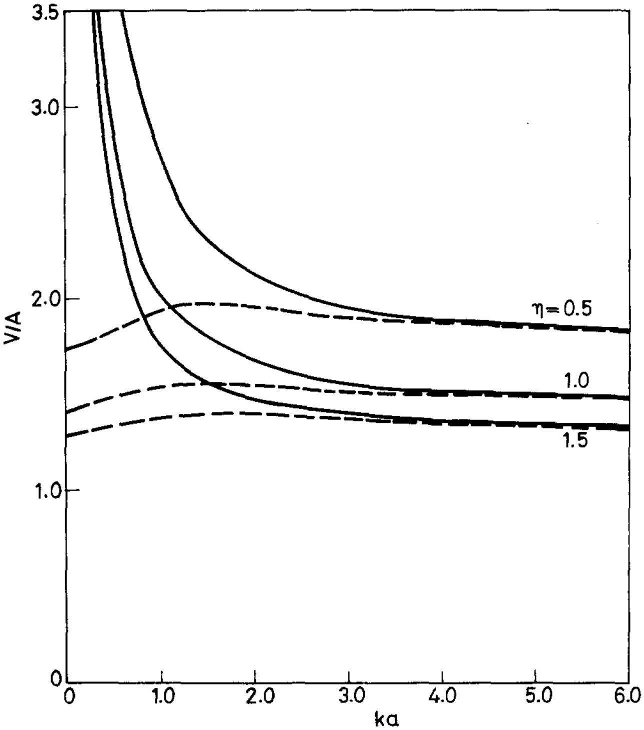

Fig. 2. Neutral curves for $m = 0$ (solid lines) and $m = 1$ (dashed lines) for different values of η.

which is the same obtained by Geronicolas (1977) for the slab geometry. From Figure 2, we see that for $ka \geq 4$, the neutral curves for $m = 0$ and $m = 1$ are coincident which leads to the stability criterion given by Equation (9). Thus for $ka \geq 4$, we conclude that the stability results for the cylindrical geometry is the same as that of the slab geometry considered by Geronicolas (1977).

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